Bachelor of Science (B.Sc.) Semester—V (C.B.S.) Examination

MATHEMATICS

(M_{10}-Metric Space, Complex Integration and Algebra)

Paper–2

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all **FIVE** questions.

(2) All questions carry equal marks.

(3) Questions **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) If $\{E_n\}$ is a sequence of countable sets, then prove that $\bigcup_{n=1}^{\infty} E_n$ is countable. 6

(B) Let X be a non-empty set. for x, y $\in$ X, define

$$d(x, y) = \begin{cases} 1, & \text{if } x \neq y \\ 0, & \text{if } x = y \end{cases}$$

Prove that d is a metric on X. 6

OR

(C) Let X be a metric space and E $\subset$ X. Prove that E is closed iff its complement is closed. 6

(D) For any collection $\{G_\alpha\}$ of open sets prove that $\bigcup_{\alpha} G\alpha$ is open. Show by an example that arbitrary intersection of open sets need not be open. 6

UNIT–II

2. (A) Let K $\subset$ Y $\subset$ X. Prove that K is compact relative to X if and only if K is compact relative to Y. 6

(B) If $\{K_\alpha\}$ is collection of compact subsets of a metric space X such that intersection of every finite subcollection of $\{k_\alpha\}$ is non-empty, then prove that $\bigcap_{\alpha} K_\alpha \neq \phi$. 6

OR

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(C) Define a complete metric space. If a sequence $\{x_n\}$ of real numbers is convergent in R, then show that $\{x_n\}$ is a Cauchy sequence in R. 6

(D) Let a subset E of the real line R^1 be connected. If $x \in E$, $y \in E$ and $x < z < y$, then show that $z \in E$. 6

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UNIT–III

3. (A) If R is a ring then prove that

(i) $a\,0 = 0\,a = 0$

(ii) $a(-b) = (-a)b = -(ab)$

(iii) $(-a)(-b) = ab$. 6

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(B) Prove that every finite integral domain is a field. 6

OR

(C) If R is a commutative ring and $a \in R$ then prove that $aR = \{ar/r \in R\}$ is an ideal of R. 6

(D) If U is an ideal of a Ring R, then prove that R/U is homomorphic to R. 6

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UNIT–IV

4. (A) Using Cauchy integral formula, calculate $\int_c \frac{dz}{z(z + \pi i)}$, where C is the circle $|z + 3i| = 1$. 6

(B) Fine the value of the integral $\int_0^{2+1} \bar{z}^2\, dz$ along the real axis from $z = 0$ to $z = 2$ and then along a line parallel to y-axis from $z = 0$ to $z = 2 + i$. 6

OR

(C) State and prove Cauchy's Residue theorem for analytic function. 6

(D) Evaluate the residues of f(z) where

$$f(z) = \frac{e^z}{z^2(z^2 + 9)} \text{ at } z = 0, -3i, 3i.$$ 6

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(Compulsory Questions)

5. (A) Prove that the set of all integers is countable. 1½

(B) Determine the limit point of the set $A = \left\{1, \frac{1}{2}, \frac{1}{3}, ...\right\}$. 1½

(C) Define K-cell and explain Z-cell. 1½

(D) Let A = (0, 1), B = (1, 2), E= A $\cup$ B. Show that E is not connected. 1½

(E) If U is an ideal of ring R and 1 $\in$ U, then prove that U = R. 1½

(F) If $\phi : R \to R'$ is a ring homomorphism, then show that $\phi(-a) = -\phi(a), \forall a \in R$. 1½

(G) Evaluate $\int_c \bar{z}\, dz$ where C is the straight line from (1, 0) to (1, 1). 1½

(H) Find zeros and poles of $\left(\frac{z+1}{z^2+1}\right)^2$. 1½

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